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Random Models of Accidents at Baltic Port and Sea Water Areas

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1. Introduction

- The concept of nonhomogeneous Poisson and nonhomogeneous compound Poisson process play a crucial role in construction of the models related to accidents on Baltic Sea water and ports play.

- Identification (estimation) of some model parameters was made based on data from reports of HELCOM [10, 11], Interreg project Baltic LINes [9], EMSA [13] and paper [7].

- The models allow us to anticipate number of accidents and accidents consequences on Baltic Sea waters and ports in future.

Table 1. Total number of ships crossing lines in the Baltic Sea during 2006-2013

Year	Passenger	Cargo	Tanker	Other	No info	Total
2006	42731	226855	67458	39627	0	376671
%	11	60	18	11	0	100
2007	43998	237740	69281	53225	8204	412448
%	11	58	17	13	2	100
2008	43060	206755	60746	104814	14689	430064
%	10	48	14	24	3	100
2009	37994	198427	68008	61014	9234	374677
%	10	53	18	16	2	100
2010	30471	181932	59409	46950	23028	342754
%	9	53	17	14	7	100
2011	35398	207273	64957	60123	23948	391699
%	9	53	17	15	6	100
2012	33193	207056	66524	54627	22959	384359
%	9	54	17	14	6	100
2013	31329	182770	61193	57959	17141	350392
%	9	52	17	17	5	100

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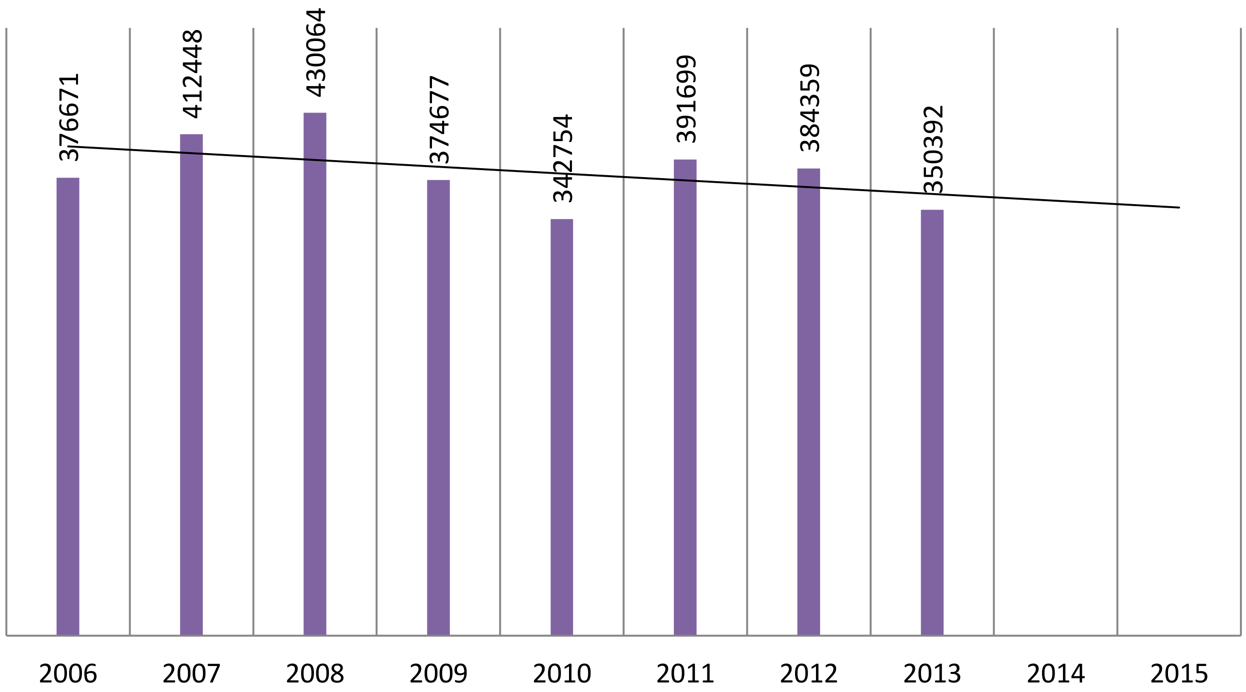

Figure 1. Total number of ships crossing in the Baltic Sea during 2006-2013

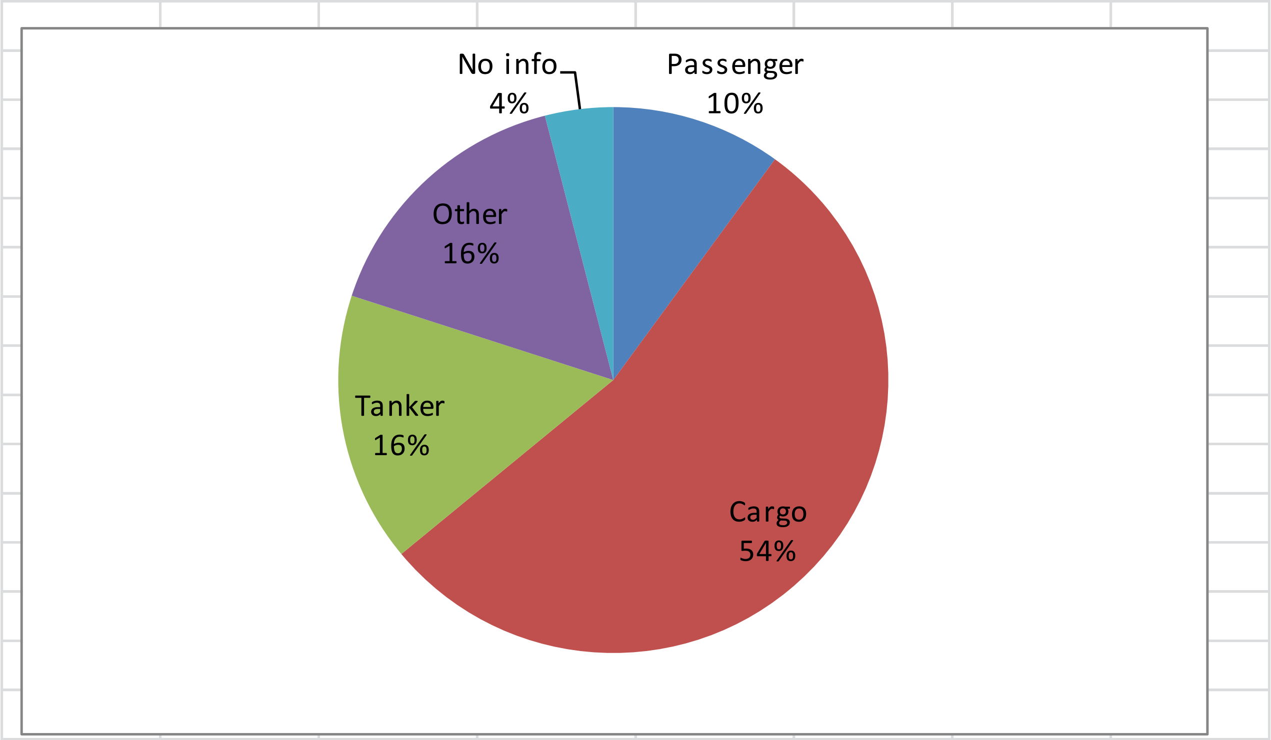

Figure 2 . Different types of cruises in ships crossing in the Baltic Sea during 2006-2013

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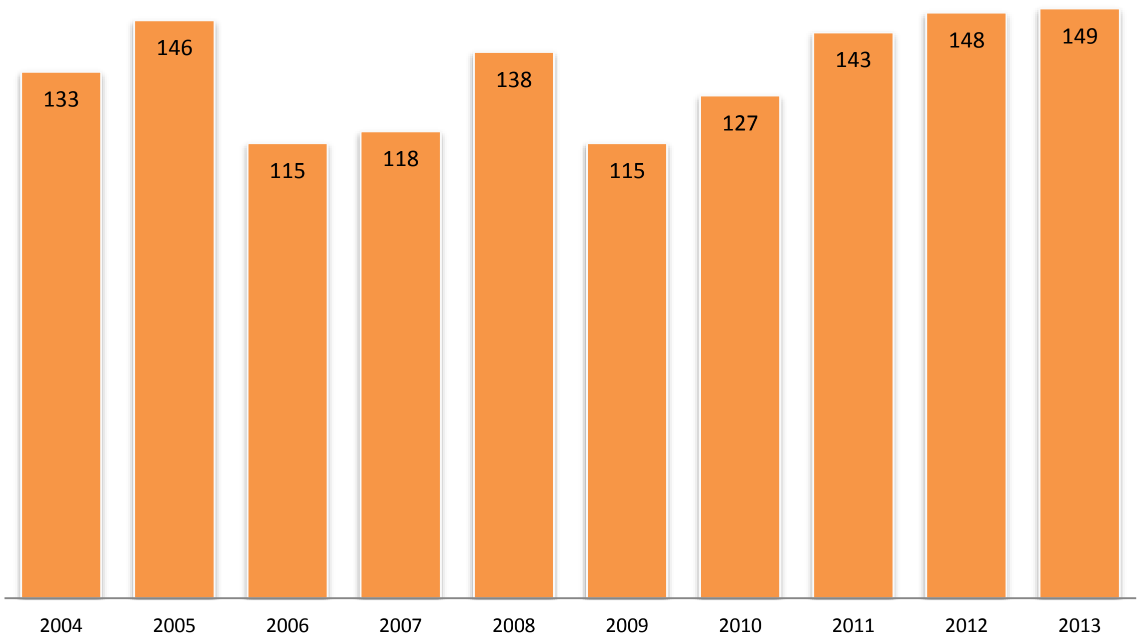

Figure 3. Total number of reporting ship accidents in the Baltic Sea during 2004-2013

We define indicators $\gamma = NSC/NA$ and $\alpha = NA/NSC$, where

NSC- number of ship crossing,

NA- number of shipping accidents.

Table 2. Indicators of shipping accidents intensity in relation to the ships crossing number

Year	$\gamma = \frac{NSC}{NA}$	$\alpha = \frac{NA}{NSC}$
2006	3275,40	0,000305
2007	3495,32	0,000286
2008	3130,84	0,000320
2009	3258,06	0,000306
2010	2698,85	0,000370
2011	2739,15	0,000365
2012	2596,97	0,000385
2013	[illegible]	0,000425

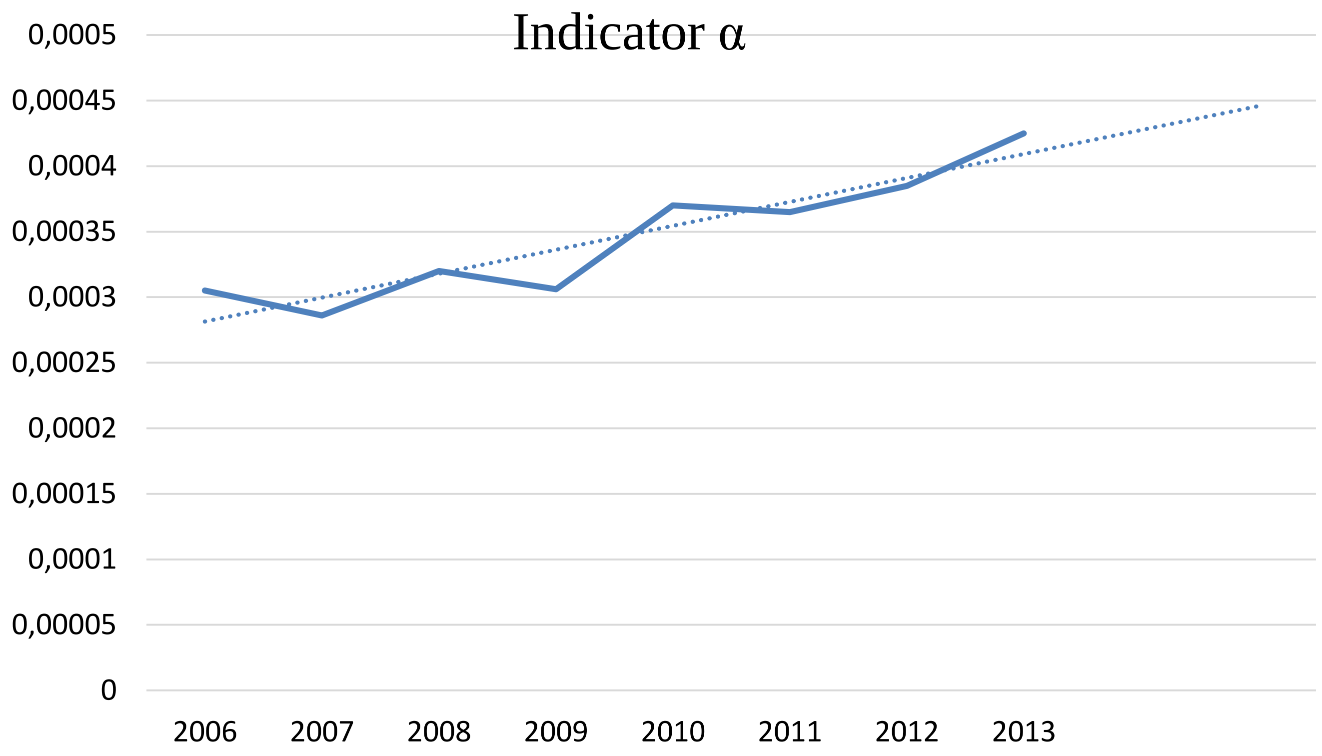

Figure 4. Indicator α of shipping accidents intensity with respect to the ships crossing numer

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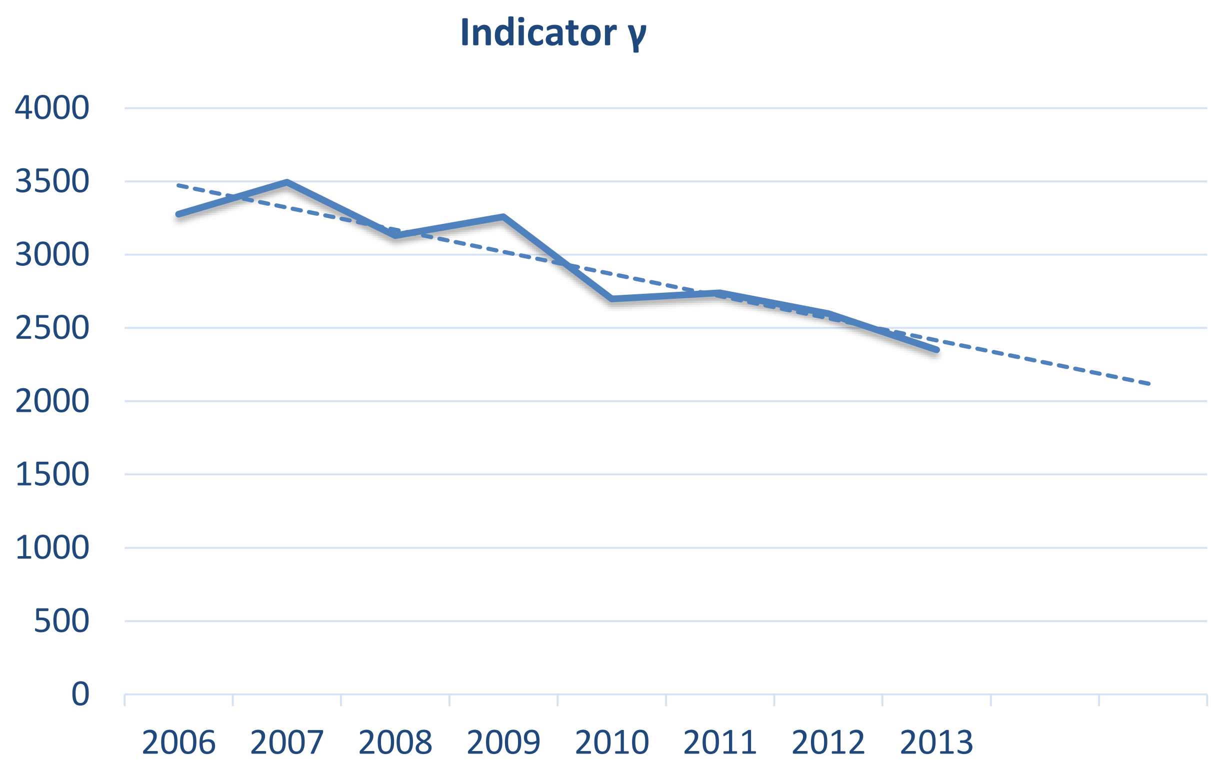

Figure 5. Indicator γ of ships crossing number with respect to the shipping accidents

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2. Nonhomogeneous Poisson process

Let $\{N(t){:}\ t \geq 0\}$ be a stochastic process taking values on $S = \{0,1,2,\ldots\}$, value of which represents the number of events in a time interval $[0, t]$.

A *counting process* $\{N(t){:}\ t \geq 0\}$ is said to be *nonhomogeneous Poisson process* (NPP) with an intensity function $\lambda(t) \geq 0,\ \ t \geq 0$, if

1. $P(N(0) = 0) = 1$; (4)

2. *The process* $\{N(t){:}\ t \geq 0\}$ *is the stochastic process with independent increments, the right continuous and piecewise constant trajectories*;

3. $P(N(t+h) - N(t) = k) = \frac{\left(\int_t^{t+h} \lambda(x)dx\right)^k}{k\,!} e^{-\int_t^{t+h} \lambda(x)dx}$; (5)

From the definition it follows that the one dimensional distribution of NPP is given by the rule

$$P(N(t)=k) = \frac{\left(\int_0^t \lambda(x)dx\right)^k}{k\,!} e^{-\int_0^t \lambda(x)dx}, \quad k = 0,1,2,\ldots \qquad (6)$$

The expectation and variance of NPP are the functions

$$\Lambda(t) = E[N(t)] = \int_0^t \lambda(x)dx \;, \qquad \mathrm{V(t)} = V[N(t)] = \int_0^t \lambda(x)dx \,, \quad t \geq 0. \qquad (7)$$

The corresponding standard deviation is

$$\mathrm{D(t)} = \sqrt{V[N(t)]} = \sqrt{\int_0^t \lambda(x)dx}, \quad t \geq 0. \qquad (8)$$

The expected value of the increment $N(t+h) - N(t)$ is

$$\Delta(t;h) = E(N(t+h) - N(t)) = \int_t^{t+h} \lambda(x)dx. \qquad (9)$$

The corresponding standard deviation is

$$\mathrm{D}(t;h) = D(N(t+h) - N(t)) = \sqrt{\int_t^{t+h} \lambda(x)dx}\;. \qquad (10)$$

3 Model of accidents number in Baltic Sea waters and ports

We will quote information from the paper [6], which is necessary for further consideration. Some mistakes in formulas (15) and (16) are noticed by author. Now this mistakes are corrected.

Assume that a stochastic process $\{N(t);\ t \geq 0\}$ taking values on $S = \{0,1,2, \ldots\}$, represents the number of accidents in the Baltic Sea and Seaports in a time interval $[0, t)$.

3.1 Assumptions

Due to the nature of these events, pre-assumption that it is a nonhomogeneous Poisson process with some parameter $\lambda(t) > 0$, seems to be justified. We can use practically these rules if will know the intensity function $\lambda(t) > 0$. To define this function we utilize information presented in [5], [9], [10, 11] The statistical analysis of the data shows that the intensity function $\lambda(t)$ can be approximated by the linear function $\lambda(t) = at + b$.

3.2 Estimation of model parameters

Dividing the number of accidents in each year, by 365 or 366 we get the intensity in units of [1 / day]. The results are shown in Table 1. We approximate the empirical intensity by a linear regression function $y = ax + b$ that satisfied condition

$$S(a, b) = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2 \rightarrow min$$

Table 3. The empirical intensity of accidents in the Baltic Sea waters and ports

Year	Interval	Centres of interval	Number of accidents	Intensity [1/day]
2004	[0, 366)	183	133	0,36338
2005	[366,731)	731,5	146	0,40000
2006	[731, 1096)	913,5	115	0,31506
2007	[1096, 1461)	1278,5	118	0,32328
2008	[1461, 1827)	1644	138	0,37704
2009	[1827, 2192)	2009,5	115	0,31506
2010	[2192, 2557)	2374,5	127	0,34794
2011	[2557, 2922)	2374,5	143	0,39178
2012	[2922, 3288)	3105	148	0,40437
2013	[3288, 3653)	3470,5	149	0,40821

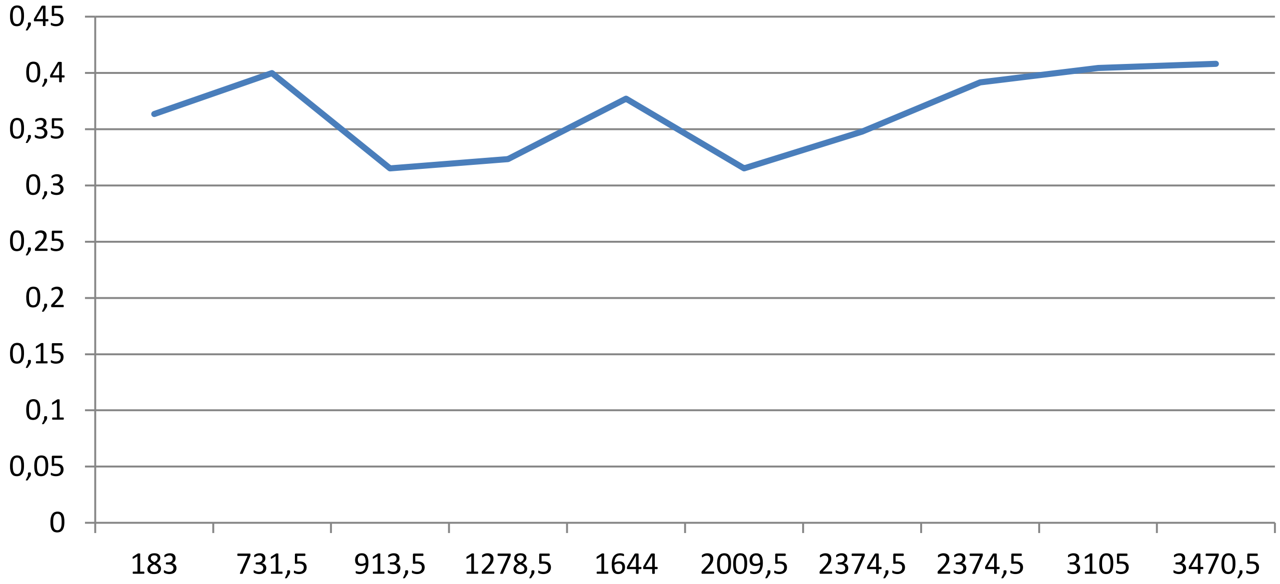

Figure 6. Empirical intensity of accidents in the Baltic Sea waters and ports

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Using Excel system we obtain

$$a = 0{,}000014756, \qquad b = 0{,}337925722. \tag{11}$$

The linear intensity of accidents is

$$\lambda(x) = 0{,}000014756\, x + 0{,}337925722, \quad x \geq 0. \tag{12}$$

From (7) we have

$$\Lambda(t) = \int_0^t (0{,}000014756\, x + 0{,}337925722) dx.$$

Hence we obtain

$$\Lambda(t) = 0{,}0000073782\, t^2 + 0{,}337925722\, t, \quad t \geq 0. \tag{13}$$

3.2 Anticipation of the accident number

Let us recall that

$$P(N(t+h)-N(t)=k)=\frac{[\Lambda(t+h)-\Lambda(t)]^k}{k\,!}e^{-[\Lambda(t+h)-\Lambda(t)]}. \qquad (14)$$

It means that we can anticipate number of accidents at any time interval with a length of h. The expected value of an increment $N(t+h)-N(t)$ is defined by (9). For the function $\Lambda(t)=\ a\,\frac{t^2}{2}+b\,t,\quad t\geq 0$ we obtain the expected value of accidents in the time interval $[t,t+h]$

$$E\big(N(t+h)-N(t)\big)=\Delta(t;h)=\ h\left(\frac{a\,h}{2}+b\ +a\,t\right). \qquad (15)$$

The corresponding standard deviation is

$$\mathrm{D}(t;h)=\sqrt{h(\frac{a\,h}{2}+b\ +a\,t)}. \qquad (16)$$

Example 1

We want to predict the number of accidents from June 1 of 2017 to August 30 of 2017. We also want to calculate the probability of a given number of accidents.

First we have to determine parameters t and h. As extension of table 2 on year 2017 we obtain an interval [4749, 5114).

From January 1 of 2017 to June 1 of 2017 have passed 151 days. Hence

$$t = 4749 + 151 = 4900.$$

From June 1 to August 31 have passed $h = 92$ days. For these parameters using (15) and (16) we obtain

$$\Delta(t; h) = 34.45, \quad \sigma(t; h) = 5.87$$

This means that the average predicted number of accidents between June 1, 2017 and August 31, 2017 is about 34 with a standard deviation of about 6. Probability that the number of accidents at the Baltic Sea waters and ports in considered interval of time is not greater than ***d***=45 and not less that ***c***=25 is

$$P_{25 \le k \le 45} = P(25 \le N(t+h) - N(t) \le 45) \ = \sum_{k=25}^{k=45} \frac{34.45^k}{k\,!} e^{-34.45};$$

Applying approximation by normal distribution we get

$$P_{25 \le k \le 45} = \Phi\left(\frac{45 - 34.45}{5.87}\right) - \Phi\left(\frac{25 - 34.45}{5.87}\right) =$$

$$= \Phi(1.7972) - \Phi(-1.6098) = 0.910.$$

3.3. Model describing number of accidents at the Baltic ports

In the article [7] reasoned that, the intensity function of the process $N_1(\mathrm{t})$ describing number of accidents at the Baltic ports is given by

$$\lambda_1(x) = 0{,}44 \times \lambda(x). \tag{17}$$

Since

$$a_1 = 0{,}44 \times 000014756 = 0{,}00000649264 \tag{18}$$

$$b_1 = 0{,}44 \times 0{,}337925722 = 0{,}14868731768, \tag{19}$$

$$\lambda_1(x) = 0{,}00000649264\, x + 0{,}14868731768 \tag{20}$$

The expected value and corresponding standard deviation of the accidents at time interval $[t, t+h)$ are

$$\Delta_1(t;h) = a_1 \frac{h^2}{2} + b_1 h + 2a_1 t h , \qquad (21)$$

$$\sigma_1(t;h) = \sqrt{a_1 \frac{h^2}{2} + b_1 h + 2 a_1 t} . \qquad (22)$$

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Example 2

We want to anticipate the number of accidents in the ports of Baltic Sea from June 1, 2017 to August 31, 2017. We calculate the probability of a given number of that kind of accidents. Parameters t and h are the same like in example. From (21) and (22) we obtain the expected value and standard deviation of accidents in ports of Baltic Sea and in the time period $[t, t+h)$.

$$\Delta_1(t;h) = 13{,}77, \quad \sigma_1(t;h) = 3{,}71$$

For example, probability that the number of accidents in the Baltic Sea Ports in this time period is not greater than ***d***=20 and not less that ***c***=10 is approximately equal to

$$P_{10 \le k \le 20} = \Phi\left(\frac{20 - 13{,}77}{3{,}71}\right) - \Phi\left(\frac{10 - 13{,}77}{3{,}71}\right) = \Phi(1{,}68) - \Phi(-1{,}02)$$
$$= 0.799$$

4. Nonhomogeneous Compound Poisson Process

e assume that $\{N(t): t \geq 0\}$ is a *nonhomogeneous Poisson process* (NPP) with an intensity function $\lambda(t) \geq 0$ for $t \geq 0$, and $X_1, X_2, \ldots$ is a sequence of the independent and identically distributed (i.i.d.) random variables independent of $\{N(t): t \geq 0\}$. A stochastic process

$$X(t) = X_1 + X_2 + \cdots + X_{N(t)}, \quad t \geq 0 \tag{23}$$

is said to be a *nonhomogeneous compound Poisson process* (NCPP).

Each of the random variables X_i describes losses in a single accident

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Corollary 1

Let $\{X(t+h)-X(t): t \geq 0\}$ be an increament of *nonhomogeneous compound Poisson process* (NCPP).

If $E(X_1^2) < \infty$, then

$$E[X(t+h)-X(t)] = \Delta(t;h)\, E(X_1), \tag{24}$$

$$V[X(t+h)-X(t)] = \Delta(t;h)\, E(X_1^2), \tag{25}$$

where

$$\Delta(t;h) = \int_t^{t+h} \lambda(x)dx. \tag{26}$$

Proposition 2

If $\{N(t): t \geq 0\}$ is a *nonhomogeneous Poisson process* (NPP) with an intensity function $\lambda(t)$, $t \geq 0$ such that $\lambda(t) \geq 0$ for $t \geq 0$, then cumulative distribution function (CDF) of the *nonhomogeneous compound*
Poisson process (NCPP) *is given by the rule*

$$G(x,t) = I_{[0,\infty)}(x)e^{-\Lambda(t)} + \sum_{k=1}^{\infty} p(k;t)F_X^{(k)}(x), \qquad (27)$$

where

$F_X^{(k)}(x)$ denotes the *k*-fold convolution of CDF of the random variables X_i, i=1,2,… and

$$p(k;t) = \frac{(\Lambda(t))^k}{k\,!}e^{-\Lambda(t)}, \; t \geq 0, \; k = 0,1,\dots, \qquad (28)$$

$$\Lambda(t) = E[N(t)] = \int_0^t \lambda(x)dx\,. \qquad (29)$$

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Corollary 3

If the random variables X_i, i=1,2,… have a discrete probability function $p_X(x) = P(X = x),\ x \in S$, then the discrete distribution function of NCPP is given by the rule

$$g(x,t) = \sum_{k=1}^{\infty} p(k;t) p_X^{(k)}(x)\,, \quad ,\ t > 0 \qquad (30)$$

where $p_X^{(k)}(x)$ denotes k –fold convolution of the discrete probability distribution $p_X(x),\ x = 0,1,2,\ldots$ of the random variable X.

5. Anticipation of the accident consequences

Let X_i, $i = 1,2,\dots,N(t)$ denotes number of fatalities or injured people or ships lost in *i*-th accident. We suppose that the random variables $X_i, i = 1,2,\dots$ have the identical Poisson distribution with parameters $E(X_i) = V(X_i) = \mu$, $i = 1,2,\dots,N(t)$.

The predicted number of fatalities in the time interval $[t, t+h)$ is described by the expectation of the increment $X(t+h) - X(t)$.

Recall that the expected value and standard deviation of the accidents number in the time interval $[t, t+h)$ are given by (15) and (16).

To calculate the expected number of fatalities in the considered time interval we apply **Proposition 2.**

Example 3

We want to anticipate the number of fatalities in accidents in the Baltic Sea waters and ports from June 1 , 2017 to August 31, 2017.

For the data from **Example 1** using (15) , (16), (24) and (25) we obtain the expected value of fatalities in the time interval $[t, t+h)$:

$$\boldsymbol{EFN} = \Delta(t;h) \times \mu \tag{31}$$

and the standard deviation

$$\boldsymbol{DFN} = \sqrt{\Delta(t;h) \times (\mu + \mu^2)} \ . \tag{32}$$

We know that the average of the sample is an unbiased estimator of the expected value. Unfortunately, reliable data are not available for the moment. We roughly estimate this parameter using data presented in EMSA reports [12, 13] and paper [7]. These data's are only partially consistent with the previous ones. The approximate estimate of the parameter μ is the number

$$\boldsymbol{\mu = 0,056} \;.$$

Applying (31) and (32) we get an expected value of fatalities and corresponding standard deviation

$$\boldsymbol{EFN = 1,9292} \quad \text{and} \quad \boldsymbol{DFN = 1,4273} \;.$$

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In this case, the formula (19) takes the form

$$g(x,t;h) = \sum_{k=1}^{\infty} \frac{\Delta(t;h)^k}{k\,!} e^{-\Delta(t;h)} \frac{(k\mu)^x}{x!} e^{-k\mu}, \quad x = 0,1,2,\ldots, \; t > 0. \qquad (33)$$

For $t = 4900$, $h = 92$ we have $\Delta(t;h) = 34.45$. Using (33), for $\mu = 0{,}056$ we obtain a predicted distribution of fatalities in accidents at the Baltic Sea and ports from June 1 , 2017 to August 31, 2017. Table 3 and figure 3 show this distribution.

We can see that the most probable numbers of fatalities are 2 and 3

Table 4. Distribution of fatalities number

x	0	1	2	3
g(*x*)	0,153175	**0,279411**	**0,262665**	0,169372
x	4	5	6	7
g(*x*)	0,0841492	0,0343135	0,0119478	0,0036499

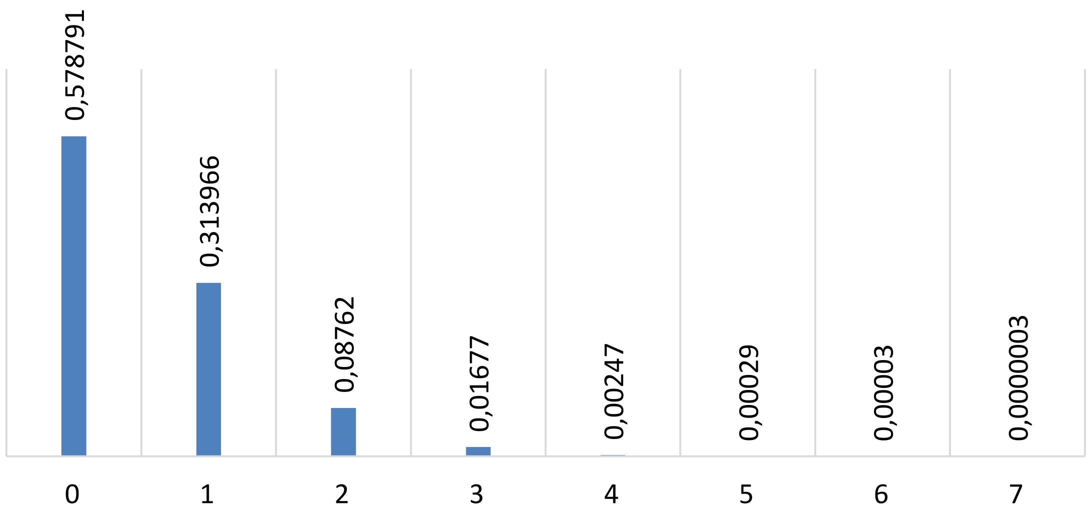

Figure 7. Distribution of fatalities number in a single accident

We can see that the most probable numbers of fatalities are 2 and 3.

Example 4

The predicted number of injured person in accidents in the Baltic Sea and Ports from June 1 2017 to August 31, 2017 we will get in a similar way. In this case

$$\boldsymbol{\mu = 0,224.}$$

For the data from **Example 3** using (31) and (32) we obtain an expected value and standard deviation of injured people number at considered period.

$$\boldsymbol{ENI} = 34{,}45 \times 0{,}224 = 7{,}7168$$

$$\boldsymbol{DNI} = \sqrt{34{,}45 \times (0{,}224 + 0{,}224^2)} = 3{,}0733$$

Equation (33) allows to compute predicted distribution of the injured person number. The results are shown in Table 4 and Figure 4.

Table 5. Distribution of injured person number

x	1	2	3	4
g(*x*)	0,0009942	0,0061322	0,0195991	0,0431724
x	5	6	7	8
g(*x*)	0,0735834	0,103326	**0,124318**	**0,131639**
x	9	10	11	12
g(*x*)	**0,125075**	0,108205	0,086211	0,063839
x	13	14	15	16
g(*x*)	0,0442636	0,0289157	0,0178902	0,0105299
x	17	18	19	20
g(*x*)	0,0059186	0,0031876	0,001649	0,0008226

Distribution of random variable X_i

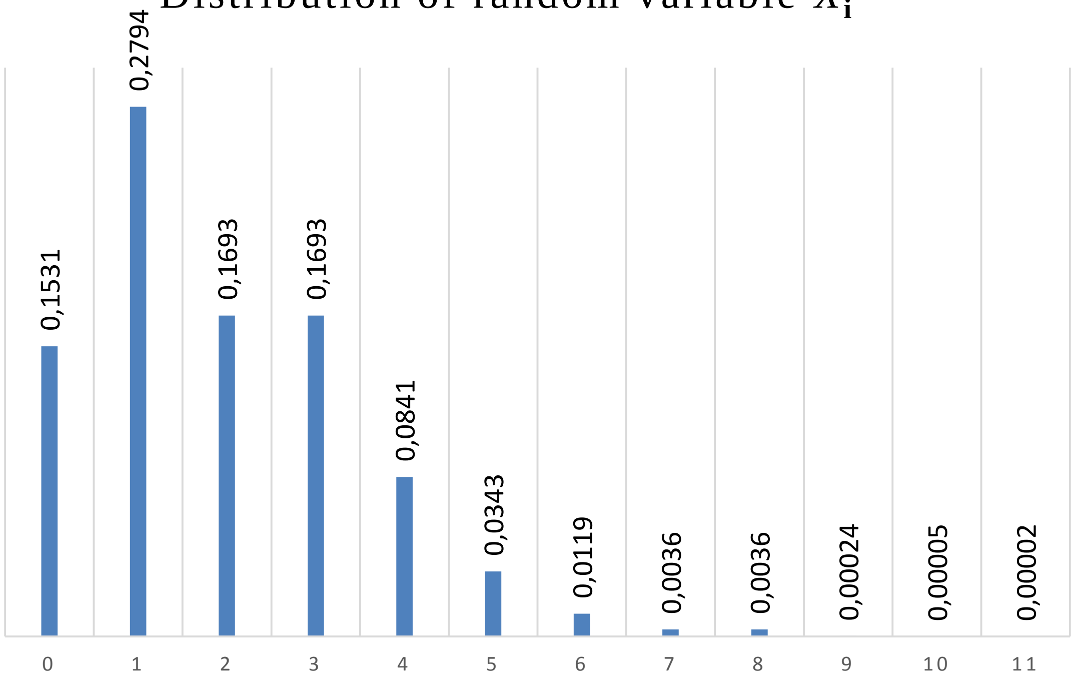

Figure 8 . Distribution of injured person number

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Example 5

For the ships lost number in accidents in the Baltic Sea and Sea Ports in considered time, interval parameter μ is

$$\boldsymbol{\mu = 0,016}\ .$$

For the data from **Example 3**, using (31) and (32) we obtain an expected value and standard deviation of the ships lost number in considered period.

$$\boldsymbol{EIN} = 34,45 \times\ 0,016 = 0,5512$$

$$\boldsymbol{DIN} = \sqrt{34,45 \times (0,016 + 0,016^2)} = 0,74834$$

Equation (33) allows to compute predicted distribution of the ship lost number. The results are shown in Table 5 and Figure 7

Table 6. Distribution of ships lost number

x	0	1	2	3
$g(x)$	0,578791	0,313966	0,0876672	0,0167734
x	4	5	6	7
$g(x)$	0,002470	0,000298	0,0000307	0,0000003

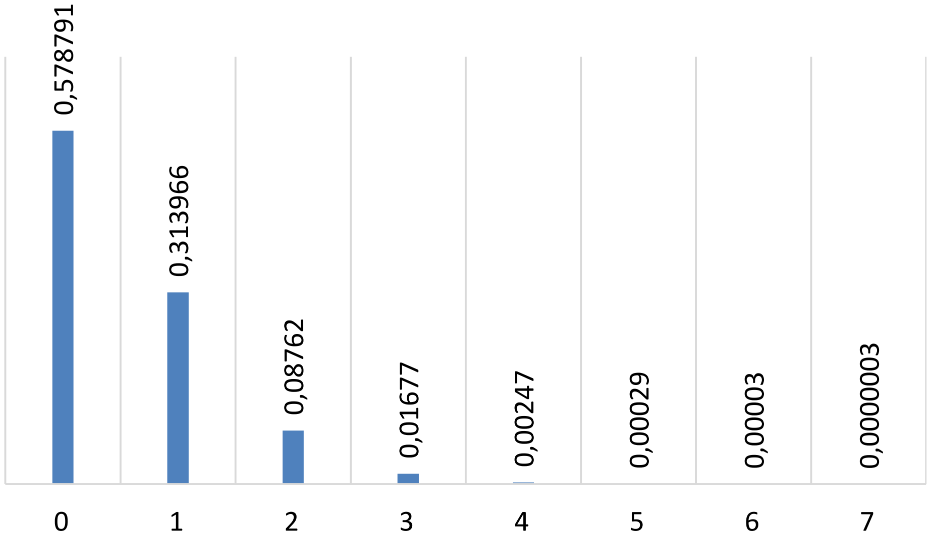

Figure 9. Distribution of ships lost number

Random parameter in Poisson model

The expected number of accidents often depends on changing randomly external conditions. Thus it can be assumed that the parameter λ is a random variable. We assume that this random variable has a gamma distribution with a density

$$f(u) = \begin{cases} \dfrac{\alpha^{\nu}}{\Gamma(\nu)} u^{\nu-1} \; e^{-\alpha u} & \text{for } \; u > 0 \\ 0 & \text{for } \; u \leq 0 \end{cases} \tag{15}$$

where $\alpha > 0, \quad \nu > 1.$

Suppose that a condition distribution of the accidents number given λ has a Poisson distribution

$$P(N(t) = k| \; \lambda) = \frac{(\lambda \, t)^{k}}{k\,!} \, e^{-\lambda \, t}, \tag{16}$$

$$k = 0, 1, 2, \ldots$$

Using the formula for the total probability is calculated unconditional one-dimensional distribution of the process

- For $k=0$ we obtain

$$P(N(t) = 0) = P(\vartheta_1 > t) = \left(\frac{\alpha}{\alpha + t}\right)^{\nu}$$

- For $k = 1, 2, \ldots$ we have

$$P(N(t) = k) = \int_0^{\infty} \frac{(ut)^k e^{-ut}}{k!} \frac{\alpha^{\nu}}{\Gamma(\nu)} u^{\nu-1} e^{-\alpha u} du \qquad (18)$$

- Finally we obtain [4]:

$$P(N(t) = k) = \frac{\nu(\nu + 1) \ldots (\nu + k - 1)}{k!} \left(\frac{t}{t + \alpha}\right)^{k} \left(\frac{\alpha}{t + \alpha}\right)^{\nu}$$

(19)

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The random variable $T = \vartheta_n$, $n = 1, 2, \ldots$ is the time which elapses between successive accidents.

The function

$$R(t) = P(T > t) = \left(\frac{\alpha}{\alpha + t}\right)^{\nu}, \qquad t \geq 0 \qquad (20)$$

is called *survival function*

The expected value and the second moment of this random variable are

$$E(T) = \int_0^{\infty} \left(\frac{\alpha}{\alpha + t}\right)^{\nu} dt = \frac{\alpha}{\nu - 1}. \qquad (23)$$

$$E(T^2) = 2\int_0^{\infty} t\left(\frac{\alpha}{\alpha + t}\right)^{\nu} dt = \frac{2\alpha^2}{(\nu - 1)(\nu - 2)} \qquad (24)$$

Procedure of parameters identification

Notice that these values depend on the two parameters: both α and ν. There is a natural question, how to determine these parameters. One method of estimating the unknown parameters is the so called the method of moments. In this method the unknown parameters are replaced by their statistical estimates derived from the results of observation. In this case, the expected value is replaced by the average of the sample and the second moment of the random variable is replaced by the second moment of the sample. Solving the corresponding system of equations we obtain the unknown parameters of the distribution.

Let the numbers $x_1, x_2, \dots, x_n$ are the values of random variables $\vartheta_1, \vartheta_2, \dots, \vartheta_n$, denoting the time between successive accidents. An estimate of the expectation $\mathrm{E(T)}$ is mean:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad (26)$$

and the estimate of is the second moment from the sample:

$$\overline{x^2} = \frac{{x_1}^2 + \cdots + {x_n}^2}{n}. \qquad (27).$$

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As a result of these calculations, we get

n= 6 , *α*= 35 , *t*= 60 , *v*= 4

- p[0]= 0.0184237
- p[1]= 0.0465442
- p[2]= 0.0734908
- p[3]= 0.0928305
- p[4]= 0.102602
- p[5]= 0.103682
- p[6]= 0.0982252
- "$P(X \leq n) = 0.5357, \quad P(X > n) = 0.4643$.

We can notice that the most probable number of accidents in a time interval length of 60 days is 5, but the probability of this event is 0.1037. The number of accidents is not greater than 6 with probability 0.5357 Probability that there will be no accidents is equal to 0.0184 whereas in the previous model, it is 0.0058. It is a three times more than the Poisson model.

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Conclusions

The random processes theory deliver concepts and theorems that enable to construct random models concerning accidents. The counting processes and processes with independent increments are the most appropriate for modelling number of the accidents number in Baltic Sea waters and ports in specified period of time. A crucial role in the models construction plays a nonhomogeneous Poisson process and nonhomogeneous compound Poisson process. Based on the nonhomogeneous Poisson process the models of accidents number in the Baltic Sea waters and Seaports have been constructed. Moreover, some procedures of the model parameters identification are presented in the paper. Estimation of model parameters was made based on data from reports of HELCOM (2014) and Interreg project Baltic LINes (2016-2019) and Herdzik paper [7]. The nonhomogeneous compound Poisson process as a model of the accidents consequences is also presented in this paper. Theoretical results are applied for anticipation the number of fatalities, number injured people and number lost ships in accidents at the Baltic Sea waters and ports in specified period of time.

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