AN ALGORITHMIC TOOL FOR SUPPORTING ROOT-CAUSE ANALYSIS OF CRITICAL INCIDENTS IN BALTIC SEA REGION PORTS

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ASSUMPTIONS REGARDING THE MODELED PORT ENVIRONMENT

1) Critical incidents that can occur in the course of operations carried out in the port area are modeled by n random processes denoted as X_i, i=1,…,n

2) m(i) different events can occur in process X_i; they are denoted as $E_a^{(i)}$, a=1,…,m(i)

3) An event can be either primary (occurring by itself) or secondary (caused by another event in the same or another process)

4) All primary events are mutually independent

5) The instances of primary event $E_a^{(i)}$ in process X_i occur according to a Poisson process with known intensity $\lambda_a^{(i)}$

ASSUMPTIONS REGARDING THE MODELED PORT ENVIRONMENT (CONT.)

6) The probability that event $E_a^{(i)}$ in process X_i directly causes event $E_b^{(j)}$ in process X_j is known for each combination of the indices i, j, a, b used for numbering the processes and events. This probability is referred to as first grade cause-effect probability and denoted as $p^{(i,j)}(a,b)$. Clearly, if no cause-effect relation exists between $E_a^{(i)}$ and $E_b^{(j)}$, then $p^{(i,j)}(a,b)=0$.

7) The intensities and first-grade cause-effect probabilities can be obtained from the available statistical data and/or by way of experts' elicitation.

8) Events can occur in a cause-effect chain called a cascade; the events in a cascade succeed each other instantaneously, thus cascades, like their initiating primary events, are mutually independent

DEFINITIONS OF CAUSE-EFFECT PROBABILITIES OF THE FIRST AND HIGHER GRADES

$p^{(i,j)}(a, b, 1)$ – the probability that event $E_a^{(i)}$ will directly cause event $E_b^{(j)}$ (first grade cause-effect probability)

$p^{(i,j)}(a, b, h)$ – the probability that event $E_a^{(i)}$ will cause event $E_b^{(j)}$ in step h, but not earlier, of a cascade triggered by $E_a^{(i)}$ (h-th grade cause-effect probability), $h \geq 2$

$p^{(i,j)}(a, b, h)$ can be regarded as the conditional probability that $E_b^{(j)}$ occurs in step h of a cascade, but not earlier, provided that the cascade is triggered by $E_a^{(i)}$

$P^{(i,j)}(a, b)$ – the probability that event $E_a^{(i)}$ will cause event $E_b^{(j)}$ in any step of a cascade triggered by $E_a^{(i)}$

$$P^{(i,j)}(a,b) = \sum_{h \geq 1} p^{(i,j)}(a,b,h) \qquad (1)$$

DEFINITIONS OF RISKS OF CRITICAL INCIDENTS

$R_b^{(j)}(k, s, t, 0)$ – the probability that exactly k instances of $E_b^{(j)}$ as a primary event occur in the time interval (s,t]

$R_b^{(j)}(k, s, t, h)$, $h \geq 1$ – the probability that exactly k instances of $E_b^{(j)}$ occur in the time interval (s, t], each in step h (but not less-than-h) of a cascade triggered by any primary event (different than $E_b^{(j)}$)

$R_b^{(j)}(k, s, t)$ – the probability that exactly k instances of $E_b^{(j)}$ (as a primary or secondary event) occur in the time interval (s, t]

DEFINITIONS OF ROOT-CAUSE PROBABILITIES

$c^{(j,i)}(b, a|h)$ – the probability that event $E_b^{(j)}$, given that it occurs in step h of a cascade, but not earlier, was caused by event $E_a^{(i)}$ that initiated that cascade ($h \geq 1$)

$c^{(j,i)}(b, a, h)$ – the probability that event $E_b^{(j)}$ occurs in step h of a cascade, but not earlier, and was caused by event $E_a^{(i)}$ that initiated that cascade ($h \geq 1$)

$C^{(j,i)}(b,a)$ – the probability that event $E_b^{(j)}$ occurs in any step $h \geq 1$ of a cascade initiated by event $E_a^{(i)}$, $(j,b) \neq (i,a)$

$C^{(j,j)}(b,b)$ – the probability that $E_b^{(j)}$ occurs as a primary event

TECHNICAL BACKGROUND – HOW TO COMPUTE THE CAUSE-EFFECT PROBABILITIES

The cause-effect probabilities of grade h, h≥2, are computed as follows:

For h≥1 we have:

$$p^{(i,i)}(a,a,h) = 0 \qquad (2)$$

while for h≥2 and (j,b) ≠ (i,a) the following recursive formula holds:

$$p^{(i,j)}(a,b,h) = \left[1 - p^{(i,j)}(a,b,1)\right] \times \coprod_{\substack{k=1,\ldots,n \\ c=1,\ldots,m(k)}} \begin{bmatrix} p^{(i,k)}(a,c,1) \times \\ p^{(k,j)}(c,b,h-1) \end{bmatrix} \qquad (3)$$

The operation „inverted pi" used in (3) is defined as follows:

$$\coprod_{k=1,,\ldots,n} x_i = 1 - \prod_{k=1,\ldots,n} (1 - x_i)$$

where $x_1,\ldots,x_n$ are numbers from the [0, 1] interval.

TECHNICAL BACKGROUND – THE MATRIX OF CAUSE-EFFECT PROBABILITIES

The behavior of the considered multi-process environment can be described by the collection of matrixes $\pi^{(i,j)}(h)$, i,j=1,…,n, h≥1, where $\pi^{(i,j)}(h)[a, b] = p^{(i,j)}(a, b, h)$ is the element in row a and column b of matrix $\pi^{(j,i)}(h)$, a=1,…,m(i), b=1,…,m(j). Thus, $\pi^{(i,j)}(h)$ expresses the impact of events occurring in process X_i on the events in process X_j, where the latter occur in step h (but not less-than-h) of cascades initiated by the former. For a fixed h, the matrixes $\pi^{(i,j)}(h)$, i,j=1,…,n, can be arranged in the matrix $\pi(h)$ as follows:

$$\pi(h) = \begin{bmatrix} \pi^{(1,1)}(h) & \pi^{(1,2)}(h) & \cdots & \pi^{(1,n)}(h) \\ \pi^{(2,1)}(h) & \pi^{(2,2)}(h) & \cdots & \pi^{(2,n)}(h) \\ \vdots & \vdots & \ddots & \vdots \\ \pi^{(n,1)}(h) & \pi^{(n,2)}(h) & \cdots & \pi^{(n,n)}(h) \end{bmatrix} \qquad (4)$$

$\pi^{(i,j)}(h)$ has m(i) rows and m(j) columns, hence $\pi(h)$ is a square matrix with $\sum_{i=1,\dots,n}$ m(i) rows and $\sum_{j=1,\dots,n}$ m(j) columns

TECHNICAL BACKGROUND – THE OPERATION $\otimes$ ON MATRICES

Let us define the matrix operation $\otimes$ as follows:

$$(A \otimes B)[q, q] = 0 \tag{6}$$

and

$$(A \otimes B)[q, r] = (1 - A[q, r])\cdot\coprod_{s=1,\ldots,\kappa(A)} A[q, s]\cdot B[s, r] \tag{7}$$

for $q \neq r$, where $\coprod$ is the "inverted pi" operation defined under formula (3). Let us note that (2) and (3) can be written as (6) and (7) if we put $A=\pi(1)$ and $B=\pi(h-1)$. We can thus convert (2) and (3) to a much simpler form:

$$\pi(h) = \pi(1) \otimes \pi(h-1) \tag{8}$$

The numerical complexity of the operations $\times$ and $\otimes$ is almost the same and formula (8) is more convenient for computer implementation than formulas (2) and (3).

TECHNICAL BACKGROUND – THE ROLE OF THE MATRIX $\pi(h)$

The matrices $\pi(h)$, $h \geq 1$, are used to easily compute the intensities of secondary events that are needed for computing the risks of critical incidents (emergencies) and root-cause probabilities of secondary incidents . The formulas for those intensities are presented in the next two slides.

FORMULAS FOR THE INTENSITIES OF SECONDARY EVENTS

The instances of $E_b^{(j)}$, such that each occurs in step h (but not less-than-h) of a cascade triggered by any primary event in any process, constitute a Poisson process with the following intensity:

$$\lambda_b^{(j)}(h) = \sum_{\substack{i=1,\dots,n \\ a=1,\dots,m(i)}} \lambda_a^{(i)} \cdot p^{(i,j)}(a,b,h) \qquad (9)$$

where (see Lemma 1) $p^{(i,j)}(a, b, h)=0$ for i=j and a=b. Let $\lambda(0)$ and $\lambda(h)$ be one-row matrices composed of the intensities $\lambda_b^{(j)}$ and $\lambda_b^{(j)}(h)$ respectively, defined as follows:

$$\lambda(0) = [\ \lambda_1^{(1)}, \dots, \lambda_{m(1)}^{(1)}; \lambda_1^{(2)}, \dots, \lambda_{m(2)}^{(2)}; \dots ; \lambda_1^{(n)}, \dots, \lambda_{m(n)}^{(n)}\ ] \qquad (10)$$

$$\lambda(h) = [\ \lambda_1^{(1)}(h),\dots, \lambda_{m(1)}^{(1)}(h); \lambda_1^{(2)}(h),\dots, \lambda_{m(2)}^{(2)}(h); \dots ; \lambda_1^{(n)}(h),\dots, \lambda_{m(n)}^{(n)}(h)\ ] \qquad (11)$$

Using the matrix $\pi(h)$, formula (9) is converted to a much shorter form:

$$\lambda(h) = \lambda(0) \times \pi(h) \qquad (12)$$

FORMULAS FOR THE INTENSITIES OF SECONDARY EVENTS (CONT.)

All instances of $E_b^{(j)}$, both primary and secondary, constitute a Poisson process with the following intensity:

$$\Lambda_b^{(j)} = \lambda_b^{(j)} + \sum_{h\geq 1} \lambda_b^{(j)}(h) \qquad (13)$$

In practice, the sum in (13) is only computed for several values of h, i.e. for $h \leq h_{max}$, where $\lambda_b^{(j)}(h)$ are negligibly small for $h > h_{max}$. Let us define the one-row matrix Λ as follows:

$$\Lambda = [\ \Lambda_1^{(1)}, \ldots, \Lambda_{m(1)}^{(1)}, \ldots, \Lambda_1^{(n)}, \ldots, \Lambda_{m(n)}^{(n)}\ ] \qquad (14)$$

Using Λ, we can convert formula (13) to the following form:

$$\Lambda = \lambda(0) + \sum_{h\geq 1} \lambda(h) \qquad (15)$$

FORMULAS FOR THE INTENSITIES OF SECONDARY EVENTS (CONT.)

A faster way of computing Λ:

$$\Lambda = \lambda(0) + \lambda(0) \times \sum_{h\geq 1} \pi(h) \qquad (16)$$

If (15) along with (12) is applied, then $\Lambda(h)$ is computed individually for each $h \leq h_{max}$, which requires h_{max} matrix multiplications. In turn, (16) only requires one matrix multiplication and $h_{max}-1$ additions, and adding $\pi(h)$ to $\pi(h+1)$ is a less complex operation than multiplying λ by $\pi(h)$. However, we need $\Lambda(h)$ if we want to compute the risks and root-cause probabilities related to step h of a cascade, as shown further.

FORMULAS FOR THE RISKS OF CRITICAL INCIDENTS

The risks defined in Slide 7 are given by the following formulas:

$$R_b^j(k,s,t,0) = \frac{\left[\lambda_b^{(j)} \cdot (t-s)\right]^k}{k!} \cdot \exp\left[-\lambda_b^{(j)} \cdot (t-s)\right] \tag{17}$$

$$R_b^j(k,s,t,h) = \frac{\left[\lambda_b^{(j)}(h) \cdot (t-s)\right]^k}{k!} \cdot \exp\left[-\lambda_b^{(j)}(h) \cdot (t-s)\right] \tag{18}$$

$$R_b^j(k,s,t) = \sum_{h\geq 0} R_b^j(k,s,t,h) = \frac{\left[\Lambda_b^{(j)} \cdot (t-s)\right]^k}{k!} \cdot \exp\left[-\Lambda_b^{(j)} \cdot (t-s)\right] \tag{19}$$

FORMULAS FOR THE ROOT-CAUSE PROBABILITIES

The root-cause probabilities defined in Slide 8 are given by the following formulas:

$$c^{(j,i)}(b,a|h) = \frac{\lambda_a^{(i)} p^{(i,j)}(a,b,h)}{\lambda_b^{(j)}(h)}, \quad h \geq 1 \tag{20}$$

$$c^{(j,i)}(b,a,h) = \lambda_a^{(i)} p^{(i,j)}(a,b,h)/\Lambda_b^{(j)}, \quad h \geq 1 \tag{21}$$

$$C^{(j,i)}(b,a) = \lambda_a^{(i)} P^{(i,j)}(a,b)/\Lambda_b^{(j)}, \ (i,a) \neq (j,b) \tag{22}$$

$$C^{(j,j)}(b,b) = \lambda_b^{(j)}/\Lambda_b^{(j)} \tag{23}$$

THE ALGORITHM COMPUTING THE RISKS AND ROOT-CAUSE PROBABILITIES

1. Arrange the input data into the matrixes $\lambda(0)$ and $\pi(1)$ defined by (10) and (4)
2. Using (8), determine the matrices $\pi(h)$, $h \geq 2$
3. Using (12), determine the matrices $\lambda(h)$, $h \geq 1$ defined by (11)
4. Using (15) or (16), determine the matrix Λ defined by (14)
5. Compute from (19) the matrix of risks $R_b^{(j)}(k, s, t)$, $j=1,\dots,n$, $b=1,\dots,m(j)$ for different k, s and t. For more detailed analysis, also compute the matrixes $R_b^{(j)}(k, s, t, h)$, $h \geq 0$ given by (17) and (18)
6. For each $E_b^{(j)}$ compute from (22) and (23) the matrix of root-cause probabilities $C^{(j,i)}(b,a)$, $i=1,\dots,n$, $a=1,\dots,m(i)$ including the probability $C^{(j,j)}(b,b)$. For more detailed analysis, also compute the matrixes $c^{(j,i)}(b,a|h)$ and $c^{(j,i)}(b,a,h)$, $h \geq 1$ given by (20) and (21)

THE ALGORITHM COMPUTING THE RISKS AND ROOT-CAUSE PROBABILITIES (CONT.)

In practice, the computations are only carried out for the first several values of h such that the elements of $\pi(h)$ differ significantly from 0. If only the probabilities $C^{(j,i)}(b,a)$ and $C^{(j,j)}(b,b)$ are to be computed, step 3 can be omitted. The author implemented the algorithm as a computer program whose example results are presented in the next slides.

A REAL-LIFE EXAMPLE

Let us consider three following processes realized in a port area including oil and container terminals:

p_1 – vessel traffic to and from the harbor, p_2 – crude oil transfer to or from tankers in the oil terminal, p_3 – truck traffic to and from the container terminal.

The following events can occur in the individual processes:

In p_1: $E_1^{(1)}$ – vessel collision with another vessel or a wharf, $E_2^{(1)}$ – spill of burning oil in the port waters, $E_3^{(1)}$ – vessel on fire; $m(1)=3$

In p_2: $E_1^{(2)}$ – pipeline or hose damage and/or ignition, $E_2^{(2)}$ – onshore tank on fire; $m(2)=2$

In p_3: $E_1^{(3)}$ – truck accident; $m(3)=1$

A REAL-LIFE EXAMPLE (CONT.)

Let us assume that the following cause-effect relations hold between the above events:

$E_1^{(1)} \rightarrow E_2^{(1)}$ (vessel collision $\rightarrow$ burning spill), $E_2^{(1)} \rightarrow E_3^{(1)}$ (burning spill $\rightarrow$ vessel on fire)

$E_2^{(1)} \rightarrow E_1^{(2)}$ (burning spill $\rightarrow$ hose ignition), $E_1^{(2)} \rightarrow E_2^{(2)}$ (hose ignition $\rightarrow$ tank on fire)

$E_2^{(2)} \rightarrow E_1^{(2)}$ (tank on fire $\rightarrow$ pipeline or hose ignition)

$E_1^{(2)} \rightarrow E_2^{(1)}$ (pipeline or hose ignition $\rightarrow$ burning spill)

$E_1^{(3)} \rightarrow E_1^{(2)}$ (truck accident $\rightarrow$ pipeline ignition)

We also assume that only $E_1^{(1)}$ (vessel collision), $E_3^{(1)}$ (vessel on fire), $E_2^{(2)}$ (tank on fire) and $E_1^{(3)}$ (truck accident) can occur as primary events; $\lambda(0) = [\ \lambda_1^{(1)}, 0, \lambda_3^{(1)}; 0, \lambda_2^{(2)}; \lambda_1^{(3)}\ ]$

However, $E_3^{(1)}$ and $E_2^{(2)}$ can be secondary events as well.

NUMERICAL RESULTS PRODUCES BY THE ALGORITHM APPLIED TO THE GIVEN EXAMPLE

INPUT DATA PRINTOUT:

Number of processes: 3

Maximum cascade grade: 6 (assumed value of h_{max})

Number of events in processes 1, 2, 3 respectively: 3, 2, 1

Matrix $\lambda[j,b]$, j=1,...,n, b=1,...,m(j):

0.5000 0.0000 0.5000

0.0000 0.5000

0.5000

Matrix $\pi(1)$:

0.0000	0.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.9000	0.4000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.9000	0.0000	0.0000	0.9000	0.0000
0.0000	0.0000	0.0000	0.5000	0.0000	0.0000
0.0000	0.0000	0.0000	0.8000	0.0000	0.0000

NUMERICAL RESULTS PRODUCES BY THE ALGORITHM APPLIED TO THE GIVEN EXAMPLE (CONT.)

RESULTS PRINTOUT; RISK MATRICES FOR DIFFERENT TIME AND QUANTITY PARAMETERS

Matrix R[j][b](2.00 years, 0 events):
0.3679 0.1782 0.0486
0.2231 0.1447
0.3679

Each of the above values subtracted from 1 is the probability that at least one respective $E_b^{(j)}$ (whether primary or not) occurs in a 2-year period.

Matrix R[j][b](2.00 years, 1 event):
0.3679 0.3073 0.1469
0.3347 0.2797
0.3679

Matrix R[j][b](2.00 years, 2 events):
0.1839 0.2651 0.2222
0.2510 0.2704
0.1839

Matrix R[j][b](2.00 years, 3 events):
0.0613 0.1524 0.2240
0.1255 0.1742
0.0613

Matrix R[j][b](2.00 years, 4 events):
0.0153 0.0657 0.1694
0.0471 0.0842
0.0153

Matrix R[j][b](2.00 years, 5 events):
0.0031 0.0227 0.1025
0.0141 0.0326
0.0031

NUMERICAL RESULTS PRODUCES BY THE ALGORITHM APPLIED TO THE GIVEN EXAMPLE (CONT.)

RESULTS PRINTOUT; MATRICES OF ROOT-CAUSE PROBABILITIES FOR DIFFERENT RESULTING EVENTS

$E_1^{(1)}$ (vessel collision) can only be a primary event

Matrix $C^{(1,i)}(2,a)$ for the resulting event $E_2^{(1)}$ (burning spill):
0.2899 0.0000 0.0000
0.0000 0.2731
0.4370

Matrix $C^{(1,i)}(3,a)$ for the resulting event $E_3^{(1)}$ (vessel on fire):
0.1567 0.0000 0.3306
0.0000 0.1972
0. 3155

Matrix $C^{(2,i)}(1,a)$ for the resulting event $E_1^{(2)}$ (pipeline or hose damage):
0.1333 0.0000 0.0000
0.0000 0.3333
0.5333

NUMERICAL RESULTS PRODUCES BY THE ALGORITHM APPLIED TO THE GIVEN EXAMPLE (CONT.)

RESULTS PRINTOUT; MATRICES OF ROOT-CAUSE PROBABILITIES FOR DIFFERENT RESULTING EVENTS

Matrix $C^{(2,i)}(2,a)$ for the resulting event $E_2^{(2)}$ (onshore tank on fire):

0.0965 0.0000 0.0000

0.0000 0.5172

0.3863

$E_1^{(3)}$ (truck accident) can only be a primary event

The green color highlites the probability that $E_b^{(j)}$, j=1,…3, b=1,…,m(j), occurs as a primary event.

END OF THE PRESENTATION

Thank you for your kind attention

Questions, suggestions and remarks are welcome